33

THE NATURE AND PROPAGATION OF LIGHT

33.1. **IDENTIFY:** For reflection, $\theta_r = \theta_a$.

SET UP: The desired path of the ray is sketched in Figure 33.1.

EXECUTE: $\tan\phi = \dfrac{14.0 \text{ cm}}{11.5 \text{ cm}}$, so $\phi = 50.6°$. $\theta_r = 90° - \phi = 39.4°$ and $\theta_r = \theta_a = 39.4°$.

EVALUATE: The angle of incidence is measured from the normal to the surface.

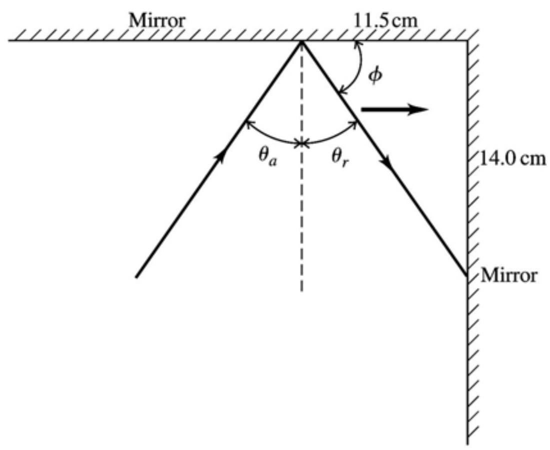


Figure 33.1

33.2. **IDENTIFY:** The speed and the wavelength of the light will be affected by the vitreous humor, but not the frequency.

SET UP: $n = \dfrac{c}{v}$. $v = f\lambda$. $\lambda = \dfrac{\lambda_0}{n}$.

EXECUTE: **(a)** $\lambda_\text{v} = \dfrac{\lambda_{0,\text{v}}}{n} = \dfrac{400 \text{ nm}}{1.34} = 299 \text{ nm}$. $\lambda_\text{r} = \dfrac{\lambda_{0,\text{r}}}{n} = \dfrac{700 \text{ nm}}{1.34} = 522 \text{ nm}$. The range is 299 nm to 522 nm.

(b) Calculate the frequency in air, where $v = c = 3.00\times10^8$ m/s. $f_\text{r} = \dfrac{c}{\lambda_\text{r}} = \dfrac{3.00\times10^8 \text{ m/s}}{700\times10^{-9} \text{ m}} = 4.29\times10^{14}$ Hz.

$f_\text{v} = \dfrac{c}{\lambda_\text{v}} = \dfrac{3.00\times10^8 \text{ m/s}}{400\times10^{-9} \text{ m}} = 7.50\times10^{14}$ Hz. The range is 4.29×10^{14} Hz to 7.50×10^{14} Hz.

(c) $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.34} = 2.24\times10^8$ m/s.

EVALUATE: The frequency range in air is the same as in the vitreous humor.

33.3. **IDENTIFY** and **SET UP:** Use Eqs. (33.1) and (33.5) to calculate v and λ.

EXECUTE: **(a)** $n = \frac{c}{v}$ so $v = \frac{c}{n} = \frac{2.998\times 10^8 \text{ m/s}}{1.47} = 2.04\times 10^8 \text{ m/s}$

(b) $\lambda = \frac{\lambda_0}{n} = \frac{650 \text{ nm}}{1.47} = 442 \text{ nm}$

EVALUATE: Light is slower in the liquid than in vacuum. By $v = f\lambda$, when v is smaller, λ is smaller.

33.4. **IDENTIFY:** In air, $c = f\lambda_0$. In glass, $\lambda = \frac{\lambda_0}{n}$.

SET UP: $c = 3.00\times 10^8 \text{ m/s}$

EXECUTE: **(a)** $\lambda_0 = \frac{c}{f} = \frac{3.00\times 10^8 \text{ m/s}}{5.80\times 10^{14} \text{ Hz}} = 517 \text{ nm}$

(b) $\lambda = \frac{\lambda_0}{n} = \frac{517 \text{ nm}}{1.52} = 340 \text{ nm}$

EVALUATE: In glass the light travels slower than in vacuum and the wavelength is smaller.

33.5. **IDENTIFY:** $n = \frac{c}{v}$. $\lambda = \frac{\lambda_0}{n}$, where λ_0 is the wavelength in vacuum.

SET UP: $c = 3.00\times 10^8$ m/s. n for air is only slightly larger than unity.

EXECUTE: **(a)** $n = \frac{c}{v} = \frac{3.00\times 10^8 \text{ m/s}}{1.94\times 10^8 \text{ m/s}} = 1.55.$

(b) $\lambda_0 = n\lambda = (1.55)(3.55\times 10^{-7} \text{ m}) = 5.50\times 10^{-7} \text{ m}.$

EVALUATE: In quartz the speed is lower and the wavelength is smaller than in air.

33.6. **IDENTIFY:** $\lambda = \frac{\lambda_0}{n}$.

SET UP: From Table 33.1, $n_{\text{water}} = 1.333$ and $n_{\text{benzene}} = 1.501$.

EXECUTE: $\lambda_{\text{water}} n_{\text{water}} = \lambda_{\text{benzene}} n_{\text{benzene}} = \lambda_0$. $\lambda_{\text{benzene}} = \lambda_{\text{water}}\left(\frac{n_{\text{water}}}{n_{\text{benzene}}}\right) = (438 \text{ nm})\left(\frac{1.333}{1.501}\right) = 389 \text{ nm}.$

EVALUATE: λ is smallest in benzene, since n is largest for benzene.

33.7. **IDENTIFY:** Apply Eqs. (33.2) and (33.4) to calculate θ_r and θ_b. The angles in these equations are measured with respect to the normal, not the surface.

(a) SET UP: The incident, reflected and refracted rays are shown in Figure 33.7.

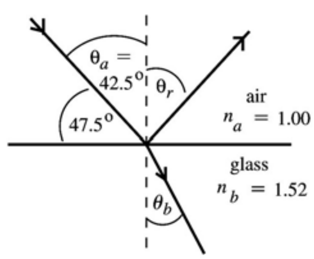


Figure 33.7

EVALUATE: $\theta_r = \theta_a = 42.5°$ The reflected ray makes an angle of $90.0° - \theta_r = 47.5°$ with the surface of the glass.

(b) $n_a \sin\theta_a = n_b \sin\theta_b$, where the angles are measured from the normal to the interface.

$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.00)(\sin 42.5°)}{1.66} = 0.4070$

$\theta_b = 24.0°$

The refracted ray makes an angle of $90.0° - \theta_b = 66.0°$ with the surface of the glass.

EVALUATE: The light is bent toward the normal when the light enters the material of larger refractive index.

33.8. **IDENTIFY:** The time delay occurs because the beam going through the transparent material travels slower than the beam in air.

SET UP: $v = \frac{c}{n}$ in the material, but $v = c$ in air.

EXECUTE: The time for the beam traveling in air to reach the detector is $t = \frac{d}{c} = \frac{2.50 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 8.33 \times 10^{-9}$ s. The light traveling in the block takes time $t = 8.33 \times 10^{-9} \text{ s} + 6.25 \times 10^{-9} \text{ s} = 1.46 \times 10^{-8}$ s. The speed of light in the block is $v = \frac{d}{t} = \frac{2.50 \text{ m}}{1.46 \times 10^{-8} \text{ s}} = 1.71 \times 10^8$ m/s. The refractive index of the block is $n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{1.71 \times 10^8 \text{ m/s}} = 1.75.$

EVALUATE: $n > 1,$ as it must be, and 1.75 is a reasonable index of refraction for a transparent material such as plastic.

33.9. **IDENTIFY** and **SET UP:** Use Snell's law to find the index of refraction of the plastic and then use Eq. (33.1) to calculate the speed v of light in the plastic.

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$

$$n_b = n_a\left(\frac{\sin\theta_a}{\sin\theta_b}\right) = 1.00\left(\frac{\sin 62.7^\circ}{\sin 48.1^\circ}\right) = 1.194$$

$$n = \frac{c}{v} \text{ so } v = \frac{c}{n} = (3.00 \times 10^8 \text{ m/s})/1.194 = 2.51 \times 10^8 \text{ m/s}$$

EVALUATE: Light is slower in plastic than in air. When the light goes from air into the plastic it is bent toward the normal.

33.10. **IDENTIFY:** Apply Snell's law at both interfaces.

SET UP: The path of the ray is sketched in Figure 33.10. Table 33.1 gives $n = 1.329$ for the methanol.

EXECUTE: **(a)** At the air-glass interface $(1.00)\sin 41.3^\circ = n_{\text{glass}} \sin\alpha.$ At the glass-methanol interface $n_{\text{glass}} \sin\alpha = (1.329)\sin\theta.$ Combining these two equations gives $\sin 41.3^\circ = 1.329 \sin\theta$ and $\theta = 29.8^\circ.$

(b) The same figure applies apply as for part (a), except $\theta = 20.2^\circ.$ $(1.00)\sin 41.3^\circ = n \sin 20.2^\circ$ and $n = 1.91.$

EVALUATE: The angle α is $25.2^\circ.$ The index of refraction of methanol is less than that of the glass and the ray is bent away from the normal at the glass $\rightarrow$ methanol interface. The unknown liquid has an index of refraction greater than that of the glass, so the ray is bent toward the normal at the glass $\rightarrow$ liquid interface.

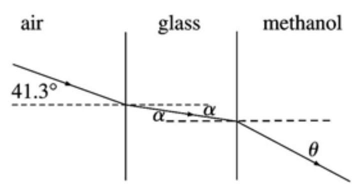


Figure 33.10

33.11. **IDENTIFY:** The figure shows the angle of incidence and angle of refraction for light going from the water into material *X*. Snell's law applies at the air-water and water-*X* boundaries.

SET UP: Snell's law says $n_a \sin\theta_a = n_b \sin\theta_b.$ Apply Snell's law to the refraction from material *X* into the water and then from the water into the air.

EXECUTE: **(a)** Material *X* to water: $n_a = n_X,$ $n_b = n_{\text{w}} = 1.333.$ $\theta_a = 25^\circ$ and $\theta_b = 48^\circ.$

$$n_a = n_b\left(\frac{\sin\theta_b}{\sin\theta_a}\right) = (1.333)\left(\frac{\sin 48^\circ}{\sin 25^\circ}\right) = 2.34.$$

(b) Water to air: As Figure 33.11 shows, $\theta_a = 48°$. $n_a = 1.333$ and $n_b = 1.00$.

$$\sin\theta_b = \left(\frac{n_a}{n_b}\right)\sin\theta_a = (1.333)\sin 48° = 82°.$$

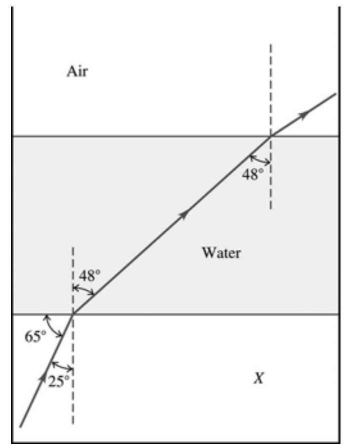


Figure 33.11

EVALUATE: $n > 1$ for material *X*, as it must be.

33.12. **IDENTIFY:** Apply Snell's law to the refraction at each interface.

SET UP: $n_{air} = 1.00$. $n_{water} = 1.333$.

EXECUTE: **(a)** $\theta_{water} = \arcsin\left(\frac{n_{air}}{n_{water}}\sin\theta_{air}\right) = \arcsin\left(\frac{1.00}{1.333}\sin 35.0°\right) = 25.5°$.

EVALUATE: **(b)** This calculation has no dependence on the glass because we can omit that step in the chain: $n_{air}\sin\theta_{air} = n_{glass}\sin\theta_{glass} = n_{water}\sin\theta_{water}$.

33.13. **IDENTIFY:** When a wave passes from one material into another, the number of waves per second that cross the boundary is the same on both sides of the boundary, so the frequency does not change. The wavelength and speed of the wave, however, do change.

SET UP: In a material having index of refraction *n*, the wavelength is $\lambda = \frac{\lambda_0}{n}$, where λ_0 is the wavelength in vacuum, and the speed is $\frac{c}{n}$.

EXECUTE: **(a)** The frequency is the same, so it is still *f*. The wavelength becomes $\lambda = \frac{\lambda_0}{n}$, so $\lambda_0 = n\lambda$. The speed is $v = \frac{c}{n}$, so $c = nv$.

(b) The frequency is still *f*. The wavelength becomes $\lambda' = \frac{\lambda_0}{n'} = \frac{n\lambda}{n'} = \left(\frac{n}{n'}\right)\lambda$ and the speed becomes

$$v' = \frac{c}{n'} = \frac{nv}{n'} = \left(\frac{n}{n'}\right)v$$

EVALUATE: These results give the speed and wavelength in a new medium in terms of the original medium without referring them to the values in vacuum (or air).

33.14. **IDENTIFY:** The wavelength of the light depends on the index of refraction of the material through which it is traveling, and Snell's law applies at the water-glass interface.

SET UP: $\lambda_0 = \lambda n$ so $\lambda_w n_w = \lambda_{gl} n_{gl}$. Snell's law gives $n_{gl}\sin\theta_{gl} = n_w \sin\theta_w$.

EXECUTE: $n_{gl} = n_w\left(\frac{\lambda_w}{\lambda_{gl}}\right) = (1.333)\left(\frac{726 \text{ nm}}{544 \text{ nm}}\right) = 1.779.$ Now apply $n_{gl}\sin\theta_{gl} = n_w \sin\theta_w$.

$\sin\theta_{gl} = \left(\frac{n_w}{n_{gl}}\right)\sin\theta_w = \left(\frac{1.333}{1.779}\right)\sin 42.0° = 0.5014.$ $\theta_{gl} = 30.1°.$

EVALUATE: $\theta_{gl} < \theta_{air}$ because $n_{gl} > n_{air}$.

33.15. **IDENTIFY:** Apply $n_a \sin\theta_a = n_b \sin\theta_b$.

SET UP: $n_a = 1.70,$ $\theta_a = 62.0°.$ $n_b = 1.58.$

EXECUTE: $\sin\theta_b = \left(\frac{n_a}{n_b}\right)\sin\theta_a = \left(\frac{1.70}{1.58}\right)\sin 62.0° = 0.950$ and $\theta_b = 71.8°.$

EVALUATE: The ray refracts into a material of smaller *n*, so it is bent away from the normal.

33.16. **IDENTIFY:** No light will enter the water if total internal reflection occurs at the glass-water boundary. Snell's law applies at the boundary.

SET UP: Find n_g, the refractive index of the glass. Then apply Snell's law at the boundary. $n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: $n_g \sin 36.2° = n_w \sin 49.8°.$ $n_g = (1.333)\left(\frac{\sin 49.8°}{\sin 36.2°}\right) = 1.724.$ Now find θ_{crit} for the glass to water refraction. $n_g \sin\theta_{crit} = n_w \sin 90.0°.$ $\sin\theta_{crit} = \frac{1.333}{1.724}$ and $\theta_{crit} = 50.6°.$

EVALUATE: For $\theta > 50.6^{o}$ at the glass-water boundary, no light is refracted into the water.

33.17. **IDENTIFY:** The critical angle for total internal reflection is θ_a that gives $\theta_b = 90°$ in Snell's law.

SET UP: In Figure 33.17 the angle of incidence θ_a is related to angle θ by $\theta_a + \theta = 90°.$

EXECUTE: **(a)** Calculate θ_a that gives $\theta_b = 90°.$ $n_a = 1.60,$ $n_b = 1.00$ so $n_a \sin\theta_a = n_b \sin\theta_b$ gives $(1.60)\sin\theta_a = (1.00)\sin 90°.$ $\sin\theta_a = \frac{1.00}{1.60}$ and $\theta_a = 38.7°.$ $\theta = 90° - \theta_a = 51.3°.$

(b) $n_a = 1.60,$ $n_b = 1.333.$ $(1.60)\sin\theta_a = (1.333)\sin 90°.$ $\sin\theta_a = \frac{1.333}{1.60}$ and $\theta_a = 56.4°.$ $\theta = 90° - \theta_a = 33.6°.$

EVALUATE: The critical angle increases when the ratio $\frac{n_a}{n_b}$ decreases.

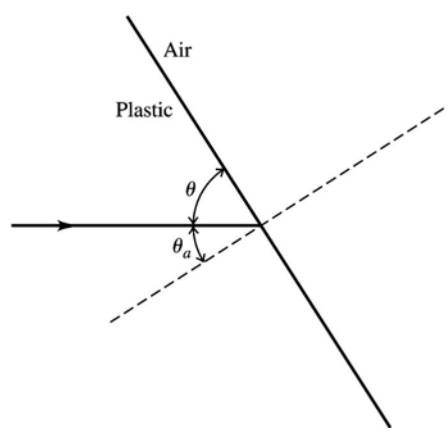


Figure 33.17

33.18. IDENTIFY: Since the refractive index of the glass is greater than that of air or water, total internal reflection will occur at the cube surface if the angle of incidence is greater than or equal to the critical angle.

SET UP: At the critical angle θ_{crit}, Snell's law gives $n_{glass}\ \sin\theta_{crit} = n_{air}\ \sin 90°$ and likewise for water.

EXECUTE: **(a)** At the critical angle θ_{crit}, $n_{glass}\ \sin\theta_{crit} = n_{air}\ \sin 90°$.

$1.53\sin\theta_{crit} = (1.00)(1)$ and $\theta_{crit} = 40.8°$.

(b) Using the same procedure as in part (a), we have $1.53\ \sin\theta_{crit} = 1.333\sin 90°$ and $\theta_{crit} = 60.6°$.

EVALUATE: Since the refractive index of water is closer to the refractive index of glass than the refractive index of air is, the critical angle for glass-to-water is greater than for glass-to-air.

33.19. IDENTIFY: Use the critical angle to find the index of refraction of the liquid.

SET UP: Total internal reflection requires that the light be incident on the material with the larger *n*, in this case the liquid. Apply $n_a\sin\theta_a = n_b\sin\theta_b$ with $a =$ liquid and $b =$ air, so $n_a = n_{liq}$ and $n_b = 1.0$.

EXECUTE: $\theta_a = \theta_{crit}$ when $\theta_b = 90°$, so $n_{liq}\sin\theta_{crit} = (1.0)\sin 90°$

$$n_{liq} = \frac{1}{\sin\theta_{crit}} = \frac{1}{\sin 42.5°} = 1.48.$$

(a) $n_a\sin\theta_a = n_b\sin\theta_b$ ($a =$ liquid, $b =$ air)

$$\sin\theta_b = \frac{n_a\sin\theta_a}{n_b} = \frac{(1.48)\sin 35.0°}{1.0} = 0.8489 \text{ and } \theta_b = 58.1°$$

(b) Now $n_a\sin\theta_a = n_b\sin\theta_b$ with $a =$ air, $b =$ liquid

$$\sin\theta_b = \frac{n_a\sin\theta_a}{n_b} = \frac{(1.0)\sin 35.0°}{1.48} = 0.3876 \text{ and } \theta_b = 22.8°$$

EVALUATE: For light traveling liquid $\rightarrow$ air the light is bent away from the normal. For light traveling air $\rightarrow$ liquid the light is bent toward the normal.

33.20. IDENTIFY: The largest angle of incidence for which any light refracts into the air is the critical angle for water $\rightarrow$ air.

SET UP: Figure 33.20 shows a ray incident at the critical angle and therefore at the edge of the circle of light. The radius of this circle is *r* and $d = 10.0$ m is the distance from the ring to the surface of the water.

EXECUTE: From the figure, $r = d\tan\theta_{crit}$. θ_{crit} is calculated from $n_a\sin\theta_a = n_b\sin\theta_b$ with $n_a = 1.333$,

$\theta_a = \theta_{crit}$, $n_b = 1.00$ and $\theta_b = 90°$. $\sin\theta_{crit} = \dfrac{(1.00)\sin 90°}{1.333}$ and $\theta_{crit} = 48.6°$.

$r = (10.0\text{ m})\tan 48.6° = 11.3\text{ m}.$

$A = \pi r^2 = \pi(11.3\text{ m})^2 = 401\text{ m}^2.$

EVALUATE: When the incident angle in the water is larger than the critical angle, no light refracts into the air.

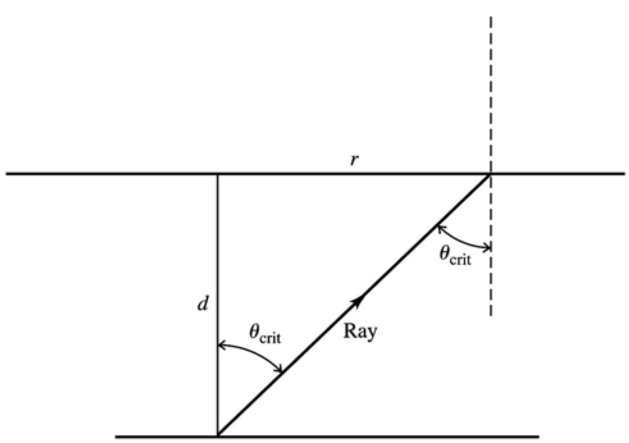


Figure 33.20

33.21. **IDENTIFY** and **SET UP:** For glass $\to$ water, $\theta_{\text{crit}} = 48.7°$. Apply Snell's law with $\theta_a = \theta_{\text{crit}}$ to calculate the index of refraction n_a of the glass.

EXECUTE: $n_a \sin\theta_{\text{crit}} = n_b \sin 90°$, so $n_a = \dfrac{n_b}{\sin\theta_{\text{crit}}} = \dfrac{1.333}{\sin 48.7°} = 1.77$

EVALUATE: For total internal reflection to occur the light must be incident in the material of larger refractive index. Our results give $n_{\text{glass}} > n_{\text{water}}$, in agreement with this.

33.22. **IDENTIFY:** If no light refracts out of the glass at the glass to air interface, then the incident angle at that interface is θ_{crit}.

SET UP: The ray has an angle of incidence of $0°$ at the first surface of the glass, so enters the glass without being bent, as shown in Figure 33.22. The figure shows that $\alpha + \theta_{\text{crit}} = 90°$.

EXECUTE: **(a)** For the glass-air interface $\theta_a = \theta_{\text{crit}}$, $n_a = 1.52$, $n_b = 1.00$ and $\theta_b = 90°$.

$n_a \sin\theta_a = n_b \sin\theta_b$ gives $\sin\theta_{\text{crit}} = \dfrac{(1.00)(\sin 90°)}{1.52}$ and $\theta_{\text{crit}} = 41.1°$. $\alpha = 90° - \theta_{\text{crit}} = 48.9°$.

(b) Now the second interface is glass $\to$ water and $n_b = 1.333$. $n_a \sin\theta_a = n_b \sin\theta_b$ gives

$\sin\theta_{\text{crit}} = \dfrac{(1.333)(\sin 90°)}{1.52}$ and $\theta_{\text{crit}} = 61.3°$. $\alpha = 90° - \theta_{\text{crit}} = 28.7°$.

EVALUATE: The critical angle increases when the air is replaced by water.

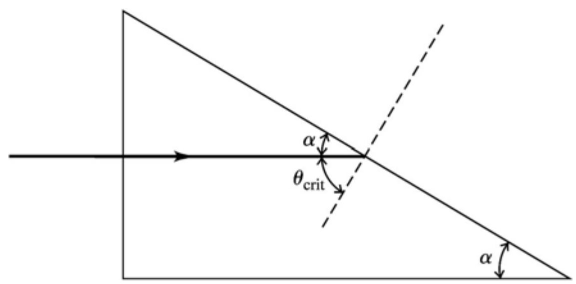


Figure 33.22

33.23. **IDENTIFY:** Total internal reflection must be occurring at the glass-water boundary. Snell's law applies there.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$. $\lambda = \lambda_0 / n$.

EXECUTE: Apply Snell's law to find n_{gl}: $n_{\text{gl}} \sin 62.0° = n_{\text{w}} \sin 90.0°$ and $n_{\text{gl}} = 1.510$. Then

$\lambda_{\text{w}} n_{\text{w}} = \lambda_{\text{gl}} n_{\text{gl}}$ and $\lambda_{\text{w}} = \lambda_{\text{gl}} \left(\dfrac{n_{\text{gl}}}{n_{\text{w}}}\right) = (408\text{ nm})\left(\dfrac{1.510}{1.333}\right) = 462\text{ nm}$.

EVALUATE: The wavelength is greater in the water than it is in the glass, as it must be, since $n_{\text{w}} < n_{\text{gl}}$.

33.24. **IDENTIFY:** We apply Snell's law to sound waves, making an appropriate definition of the index of refraction for sound. We cannot use the speed of sound in vacuum because sound does not travel through a vacuum.

SET UP: $n = \dfrac{v_{\text{air}}}{v}$. When $\theta_a = \theta_{\text{crit}}$, $\theta_b = 90°$. $n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: **(a)** For air, $n = \dfrac{v_{\text{air}}}{v} = 1.00$. For water, $n = \dfrac{v_{\text{air}}}{v} = \dfrac{344\text{ m/s}}{1320\text{ m/s}} = 0.261$. Air has a larger index of refraction for sound waves.

(b) Total internal reflection requires that the waves be incident in the material of larger refractive index. $n_a = 1.00$, $n_b = 0.261$, $\theta_a = \theta_{\text{crit}}$, and $\theta_b = 90°$. Applying $n_a \sin\theta_a = n_b \sin\theta_b$ gives

$$\sin\theta_{\text{crit}} = \left(\frac{0.261}{1.00}\right)\sin 90°, \text{ so } \theta_{\text{crit}} = 15.1°.$$

(c) The sound wave must be traveling in air.

(d) Sound waves can be totally reflected from the surface of the water.

EVALUATE: Light travels faster in vacuum than in any material and *n* is always greater than 1.00. Sound travels faster in solids and liquids than in air and *n* for sound is less than 1.00.

33.25. **IDENTIFY:** The index of refraction depends on the wavelength of light, so the light from the red and violet ends of the spectrum will be bent through different angles as it passes into the glass. Snell's law applies at the surface.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$. From the graph in Figure 33.18 in the textbook, for $\lambda = 400$ nm (the violet end of the visible spectrum), $n = 1.67$ and for $\lambda = 700$ nm (the red end of the visible spectrum), $n = 1.62$. The path of a ray with a single wavelength is sketched in Figure 33.25.

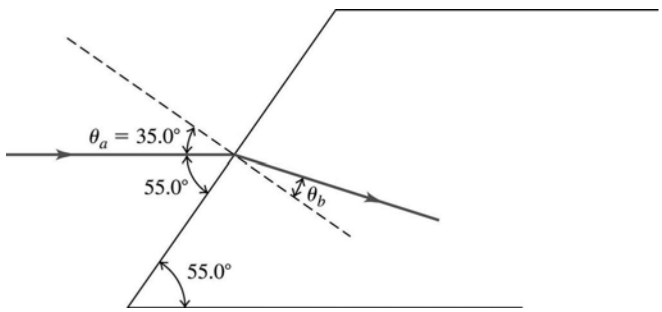


Figure 33.25

EXECUTE: For $\lambda = 400$ nm, $\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a = \frac{1.00}{1.67}\sin 35.0°$, so $\theta_b = 20.1°$. For $\lambda = 700$ nm,

$$\sin\theta_b = \frac{1.00}{1.62}\sin 35.0°, \text{ so } \theta_b = 20.7°. \ \Delta\theta \text{ is about } 0.6°.$$

EVALUATE: This angle is small, but the separation of the beams could be fairly large if the light travels through a fairly large slab.

33.26. **IDENTIFY:** Snell's law is $n_a \sin\theta_a = n_b \sin\theta_b$. $v = \frac{c}{n}$.

SET UP: a = air, b = glass.

EXECUTE: **(a)** red: $n_b = \frac{n_a \sin\theta_a}{\sin\theta_b} = \frac{(1.00)\sin 57.0°}{\sin 38.1°} = 1.36$. violet: $n_b = \frac{(1.00)\sin 57.0°}{\sin 36.7°} = 1.40$.

(b) red: $v = \frac{c}{n} = \frac{3.00\times10^8 \text{ m/s}}{1.36} = 2.21\times10^8$ m/s; violet: $v = \frac{c}{n} = \frac{3.00\times10^8 \text{ m/s}}{1.40} = 2.14\times10^8$ m/s.

EVALUATE: *n* is larger for the violet light and therefore this light is bent more toward the normal, and the violet light has a smaller speed in the glass than the red light.

33.27. **IDENTIFY:** The first polarizer filters out half the incident light. The fraction filtered out by the second polarizer depends on the angle between the axes of the two filters.

SET UP: $I = I_0 \cos^2\phi$.

EXECUTE: After the first filter, $I = \frac{1}{2}I_0$. After the second filter, $I = \left(\frac{1}{2}I_0\right)\cos^2\phi$, which gives

$$I = \left(\frac{1}{2}I_0\right)\cos^2 30.0° = 0.375 I_0.$$

EVALUATE: The only variable that affects the answer is the angle between the axes of the two polarizers.

33.28. **IDENTIFY:** The sunlight must be striking the lake surface at the Brewster's angle (the polarizing angle) since the reflected light is completely polarized.

SET UP: The reflected beam is completely polarized when $\theta_a = \theta_p$, with $\tan\theta_p = \dfrac{n_b}{n_a}$. $n_a = 1.00$, $n_b = 1.333$. θ_p is measured relative to the normal to the surface.

EXECUTE: **(a)** $\tan\theta_p = \dfrac{1.333}{1.00}$, so $\theta_p = 53.1°$. The sunlight is incident at an angle of $90° - 53.1° = 36.9°$ above the horizontal.

(b) Figure 33.27 in the text shows that the plane of the electric field vector in the reflected light is horizontal.

EVALUATE: To reduce the glare (intensity of reflected light), sunglasses with polarizing filters should have the filter axis vertical.

33.29. **IDENTIFY:** When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity I_{max} is incident on a polarizer, the transmitted intensity is $I = I_{max}\cos^2\phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

SET UP: For the second polarizer $\phi = 60°$. For the third polarizer, $\phi = 90° - 60° = 30°$.

EXECUTE: **(a)** At point *A* the intensity is $I_0/2$ and the light is polarized along the vertical direction. At point *B* the intensity is $(I_0/2)(\cos 60°)^2 = 0.125 I_0$, and the light is polarized along the axis of the second polarizer. At point *C* the intensity is $(0.125 I_0)(\cos 30°)^2 = 0.0938 I_0$.

(b) Now for the last filter $\phi = 90°$ and $I = 0$.

EVALUATE: Adding the middle filter increases the transmitted intensity.

33.30. **IDENTIFY:** Apply Snell's law.

SET UP: The incident, reflected and refracted rays are shown in Figure 33.30.

EXECUTE: From the figure, $\theta_b = 37.0°$ and $n_b = n_a \dfrac{\sin\theta_a}{\sin\theta_b} = 1.33\dfrac{\sin 53°}{\sin 37°} = 1.77$.

EVALUATE: The refractive index of *b* is greater than that of *a*, and the ray is bent toward the normal when it refracts.

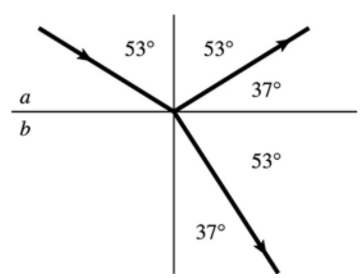


Figure 33.30

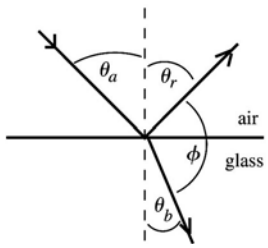
33.31. **IDENTIFY** and **SET UP:** Reflected beam completely linearly polarized implies that the angle of incidence equals the polarizing angle, so $\theta_p = 54.5°$. Use Eq. (33.8) to calculate the refractive index of the glass. Then use Snell's law to calculate the angle of refraction.

EXECUTE: **(a)** $\tan\theta_p = \dfrac{n_b}{n_a}$ gives $n_{glass} = n_{air}\tan\theta_p = (1.00)\tan 54.5° = 1.40$.

(b) $n_a \sin\theta_a = n_b \sin\theta_b$

$$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.00)\sin 54.5°}{1.40} = 0.5815 \text{ and } \theta_b = 35.5°$$

EVALUATE:



Note: $\phi = 180.0° - \theta_r - \theta_b$ and $\theta_r = \theta_a$. Thus $\phi = 180.0° - 54.5° - 35.5° = 90.0°$; the reflected ray and the refracted ray are perpendicular to each other. This agrees with Figure 33.28 in the text book.

Figure 33.31

33.32. **IDENTIFY:** Set $I = I_0/10$, where I is the intensity of light passed by the second polarizer.

SET UP: When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity I_{max} is incident on a polarizer, the transmitted intensity is $I = I_{\text{max}} \cos^2\phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

EXECUTE: **(a)** After the first filter $I = \dfrac{I_0}{2}$ and the light is polarized along the vertical direction. After the second filter we want $I = \dfrac{I_0}{10}$, so $\dfrac{I_0}{10} = \left(\dfrac{I_0}{2}\right)(\cos\phi)^2$. $\cos\phi = \sqrt{2/10}$ and $\phi = 63.4°$.

(b) Now the first filter passes the full intensity I_0 of the incident light. For the second filter $\dfrac{I_0}{10} = I_0(\cos\phi)^2$. $\cos\phi = \sqrt{1/10}$ and $\phi = 71.6°$.

EVALUATE: When the incident light is polarized along the axis of the first filter, ϕ must be larger to achieve the same overall reduction in intensity than when the incident light is unpolarized.

33.33. **IDENTIFY:** From Malus's law, the intensity of the emerging light is proportional to the *square* of the cosine of the angle between the polarizing axes of the two filters.

SET UP: If the angle between the two axes is θ, the intensity of the emerging light is $I = I_{\text{max}} \cos^2\theta$.

EXECUTE: At angle $\theta, I = I_{\text{max}} \cos^2\theta$, and at the new angle α, $\frac{1}{2}I = I_{\text{max}} \cos^2\alpha$. Taking the ratio of the intensities gives $\dfrac{I_{\text{max}} \cos^2\alpha}{I_{\text{max}} \cos^2\theta} = \dfrac{\frac{1}{2}I}{I}$, which gives us $\cos\alpha = \dfrac{\cos\theta}{\sqrt{2}}$. Solving for α yields $\alpha = \arccos\left(\dfrac{\cos\theta}{\sqrt{2}}\right)$.

EVALUATE: For $\theta = 0°$, $I = I_{\text{max}}$. The expression we derived then gives $\alpha = 45°$ and for this angle between the axes of the two filters, $I = I_{\text{max}}/2$. So, our expression is seen to be correct for this special case.

33.34. **IDENTIFY:** The reflected light is completely polarized when the angle of incidence equals the polarizing angle θ_{p}, where $\tan\theta_{\text{p}} = \dfrac{n_b}{n_a}$.

SET UP: $n_b = 1.66$.

EXECUTE: **(a)** $n_a = 1.00$. $\tan\theta_{\text{p}} = \dfrac{1.66}{1.00}$ and $\theta_{\text{p}} = 58.9°$.

(b) $n_a = 1.333$. $\tan\theta_{\text{p}} = \dfrac{1.66}{1.333}$ and $\theta_{\text{p}} = 51.2°$.

EVALUATE: The polarizing angle depends on the refractive indicies of both materials at the interface.

33.35. **IDENTIFY:** When unpolarized light of intensity I_0 is incident on a polarizing filter, the transmitted light has intensity $\frac{1}{2}I_0$ and is polarized along the filter axis. When polarized light of intensity I_0 is incident on a polarizing filter the transmitted light has intensity $I_0 \cos^2\phi$.

SET UP: For the second filter, $\phi = 62.0° - 25.0° = 37.0°$.

EXECUTE: After the first filter the intensity is $\frac{1}{2}I_0 = 10.0\ \text{W/m}^2$ and the light is polarized along the axis of the first filter. The intensity after the second filter is $I = I_0\cos^2\phi$, where $I_0 = 10.0\ \text{W/m}^2$ and $\phi = 37.0°$. This gives $I = 6.38\ \text{W/m}^2$.

EVALUATE: The transmitted intensity depends on the angle between the axes of the two filters.

33.36. **IDENTIFY:** Use the transmitted intensity when all three polairzers are present to solve for the incident intensity I_0. Then repeat the calculation with only the first and third polarizers.

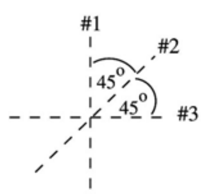
SET UP: For unpolarized light incident on a filter, $I = \frac{1}{2}I_0$ and the light is linearly polarized along the filter axis. For polarized light incident on a filter, $I = I_{\max}(\cos\phi)^2$, where $I_{\max}$ is the intensity of the incident light, and the emerging light is linearly polarized along the filter axis.

EXECUTE: With all three polarizers, if the incident intensity is I_0 the transmitted intensity is $I = (\frac{1}{2}I_0)(\cos 23.0°)^2(\cos[62.0° - 23.0°])^2 = 0.256I_0$. $I_0 = \dfrac{I}{0.256} = \dfrac{75.0\ \text{W/cm}^2}{0.256} = 293\ \text{W/cm}^2$. With only the first and third polarizers, $I = (\frac{1}{2}I_0)(\cos 62.0°)^2 = 0.110I_0 = (0.110)(293\ \text{W/cm}^2) = 32.2\ \text{W/cm}^2$.

EVALUATE: The transmitted intensity is greater when all three filters are present.

33.37. **IDENTIFY** and **SET UP:** Apply Eq. (33.7) to polarizers #2 and #3. The light incident on the first polarizer is unpolarized, so the transmitted light has half the intensity of the incident light, and the transmitted light is polarized.

(a) EXECUTE: The axes of the three filters are shown in Figure 33.37a.

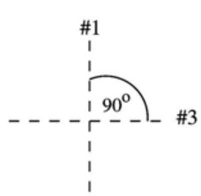


$I = I_{\max}\cos^2\phi$

Figure 33.37a

After the first filter the intensity is $I_1 = \frac{1}{2}I_0$ and the light is linearly polarized along the axis of the first polarizer. After the second filter the intensity is $I_2 = I_1\cos^2\phi = (\frac{1}{2}I_0)(\cos 45.0°)^2 = 0.250I_0$ and the light is linearly polarized along the axis of the second polarizer. After the third filter the intensity is $I_3 = I_2\cos^2\phi = 0.250I_0(\cos 45.0°)^2 = 0.125I_0$ and the light is linearly polarized along the axis of the third polarizer.

(b) The axes of the remaining two filters are shown in Figure 33.37b.



After the first filter the intensity is $I_1 = \frac{1}{2}I_0$ and the light is linearly polarized along the axis of the first polarizer.

Figure 33.37b

After the next filter the intensity is $I_3 = I_1 \cos^2\phi = \left(\tfrac{1}{2}I_0\right)(\cos 90.0°)^2 = 0.$ No light is passed.

EVALUATE: Light is transmitted through all three filters, but no light is transmitted if the middle polarizer is removed.

33.38. **IDENTIFY:** The shorter the wavelength of light, the more it is scattered. The intensity is inversely proportional to the fourth power of the wavelength.

SET UP: The intensity of the scattered light is proportional to $1/\lambda^4$; we can write it as $I = (\text{constant})/\lambda^4.$

EXECUTE: **(a)** Since I is proportional to $1/\lambda^4$, we have $I = (\text{constant})/\lambda^4.$ Taking the ratio of the intensity of the red light to that of the green light gives

$$\frac{I_R}{I} = \frac{(\text{constant})/\lambda_R^4}{(\text{constant})/\lambda_G^4} = \left(\frac{\lambda_G}{\lambda_R}\right)^4 = \left(\frac{520 \text{ nm}}{665 \text{ nm}}\right)^4 = 0.374, \text{ so } I_R = 0.374I.$$

(b) Following the same procedure as in part (a) gives $\frac{I_V}{I} = \left(\frac{\lambda_G}{\lambda_V}\right)^4 = \left(\frac{520 \text{ nm}}{420 \text{ nm}}\right)^4 = 2.35, \text{ so } I_V = 2.35I.$

EVALUATE: In the scattered light, the intensity of the short-wavelength violet light is about 7 times as great as that of the red light, so this scattered light will have a blue-violet color.

33.39. **IDENTIFY:** Reflection reverses the sign of the component of light velocity perpendicular to the reflecting surface but leaves the other components unchanged.

SET UP: Consider three mirrors, M_1 in the (x,y)-plane, M_2 in the (y,z)-plane and M_3 in the (x,z)-plane.

EXECUTE: A light ray reflecting from M_1 changes the sign of the z-component of the velocity, reflecting from M_2 changes the x-component and from M_3 changes the y-component. Thus the velocity, and hence also the path, of the light beam flips by 180°.

EVALUATE: Example 33.3 discusses some uses of corner reflectors.

33.40. **IDENTIFY:** The light travels slower in the jelly than in the air and hence will take longer to travel the length of the tube when it is filled with jelly than when it contains just air.

SET UP: The definition of the index of refraction is $n = c/v$, where v is the speed of light in the jelly.

EXECUTE: First get the length L of the tube using air. In the air, we have $L = ct = (3.00\times 10^8 \text{ m/s})(8.72 \text{ ns}) = 2.616 \text{ m}.$ The speed in the jelly is

$$v = \frac{L}{t} = (2.616 \text{ m})/(8.72 \text{ ns} + 2.04 \text{ ns}) = 2.431\times 10^8 \text{ m/s}. \quad n = \frac{c}{v} = (3.00\times 10^8 \text{ m/s})/(2.431\times 10^8 \text{ m/s}) = 1.23$$

EVALUATE: A high-speed timer would be needed to measure times as short as a few nanoseconds.

33.41. **IDENTIFY:** Snell's law applies to the sound waves in the heart. (See Exercise 33.24.)

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b.$ If θ_a is the critical angle then $\theta_b = 90°.$ For air, $n_{\text{air}} = 1.00.$ For heart muscle, $n_{\text{mus}} = \frac{344 \text{ m/s}}{1480 \text{ m/s}} = 0.2324.$

EXECUTE: **(a)** $n_a \sin\theta_a = n_b \sin\theta_b$ gives $(1.00)\sin(9.73°) = (0.2324)\sin\theta_b.$ $\sin\theta_b = \frac{\sin(9.73°)}{0.2324}$ so $\theta_b = 46.7°.$

(b) $(1.00)\sin\theta_{\text{crit}} = (0.2324)\sin 90°$ gives $\theta_{\text{crit}} = 13.4°.$

EVALUATE: To interpret a sonogram, it should be important to know the true direction of travel of the sound waves within muscle. This would require knowledge of the refractive index of the muscle.

33.42. **IDENTIFY:** Use the change in transit time to find the speed v of light in the slab, and then apply $n = \frac{c}{v}$ and $\lambda = \frac{\lambda_0}{n}.$

SET UP: It takes the light an additional 4.2 ns to travel 0.840 m after the glass slab is inserted into the beam.

EXECUTE: $\dfrac{0.840\text{ m}}{c/n}-\dfrac{0.840\text{ m}}{c}=(n-1)\dfrac{0.840\text{ m}}{c}=4.2\text{ ns}.$ We can now solve for the index of refraction: $n=\dfrac{(4.2\times10^{-9}\text{s})(3.00\times10^{8}\text{m/s})}{0.840\text{ m}}+1=2.50.$ The wavelength inside of the glass is $\lambda=\dfrac{490\text{ nm}}{2.50}=196\text{ nm}.$

EVALUATE: Light travels slower in the slab than in air and the wavelength is shorter.

33.43. **IDENTIFY:** The angle of incidence at *A* is to be the critical angle. Apply Snell's law at the air to glass refraction at the top of the block.

SET UP: The ray is sketched in Figure 33.43.

EXECUTE: For glass $\rightarrow$ air at point *A*, Snell's law gives $(1.38)\sin\theta_{\text{crit}}=(1.00)\sin 90°$ and $\theta_{\text{crit}}=46.4°.$ $\theta_b=90°-\theta_{\text{crit}}=43.6°.$ Snell's law applied to the refraction from air to glass at the top of the block gives $(1.00)\sin\theta_a=(1.38)\sin(43.6°)$ and $\theta_a=72.1°.$

EVALUATE: If θ_a is larger than $72.1°$ then the angle of incidence at point *A* is less than the initial critical angle and total internal reflection doesn't occur.

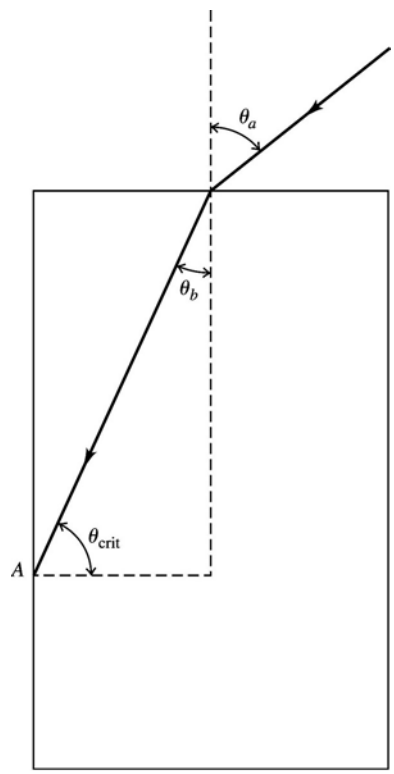


Figure 33.43

33.44. **IDENTIFY:** As the light crosses the glass-air interface along *AB*, it is refracted and obeys Snell's law.

SET UP: Snell's law is $n_a\sin\theta_\text{a}=n_b\sin\theta_\text{b}$ and $n=1.000$ for air. At point *B* the angle of the prism is $30.0°$.

EXECUTE: Apply Snell's law at *AB*. The prism angle at *A* is 60.0°, so for the upper ray, the angle of incidence at *AB* is $60.0°+12.0°=72.0°.$ Using this value gives $n_1\sin 60.0°=\sin 72.0°$ and $n_1=1.10.$ For the lower ray, the angle of incidence at *AB* is $60.0°+12.0°+8.50°=80.5°,$ giving $n_2\sin 60.0°=\sin 80.5°$ and $n_2=1.14.$

EVALUATE: The lower ray is deflected more than the upper ray because that wavelength has a slightly greater index of refraction than the upper ray.

33.45. **IDENTIFY:** For total internal reflection, the angle of incidence must be at least as large as the critical angle.

SET UP: The angle of incidence for the glass-oil interface must be the critical angle, so $\theta_b = 90°$.

$n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$ gives $(1.52)\sin 57.2° = n_{\text{oil}} \sin 90°$. $n_{\text{oil}} = (1.52)\sin 57.2° = 1.28$.

EVALUATE: $n_{\text{oil}} > 1$, which it must be, and 1.28 is a reasonable value for an oil.

33.46. **IDENTIFY:** Apply $\lambda = \dfrac{\lambda_0}{n}$. The number of wavelengths in a distance *d* of a material is $\dfrac{d}{\lambda}$ where λ is the wavelength in the material.

SET UP: The distance in glass is $d_{\text{glass}} = 0.00250 \text{ m}$. The distance in air is

$d_{\text{air}} = 0.0180 \text{ m} - 0.00250 \text{ m} = 0.0155 \text{ m}$.

EXECUTE: number of wavelengths $=$ number in air $+$ number in glass.

$$\text{number of wavelengths} = \frac{d_{\text{air}}}{\lambda} + \frac{d_{\text{glass}}}{\lambda} n = \frac{0.0155 \text{ m}}{5.40 \times 10^{-7} \text{ m}} + \frac{0.00250 \text{ m}}{5.40 \times 10^{-7} \text{ m}}(1.40) = 3.52 \times 10^4.$$

EVALUATE: Without the glass plate the number of wavelengths between the source and screen is $\dfrac{0.0180 \text{ m}}{5.40 \times 10^{-3} \text{ m}} = 3.33 \times 10^4$. The wavelength is shorter in the glass so there are more wavelengths in a distance in glass than there are in the same distance in air.

33.47. **IDENTIFY:** Find the critical angle for glass $\rightarrow$ air. Light incident at this critical angle is reflected back to the edge of the halo.

SET UP: The ray incident at the critical angle is sketched in Figure 33.47.

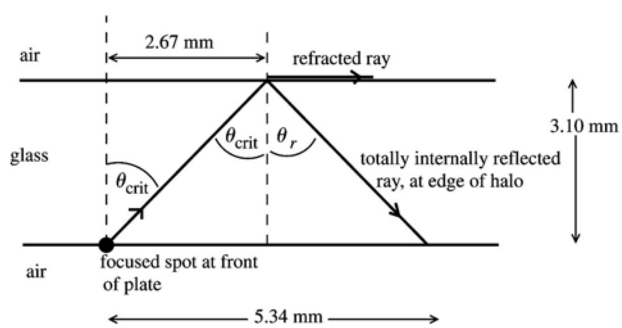


Figure 33.47

EXECUTE: From the distances given in the sketch, $\tan\theta_{\text{crit}} = \dfrac{2.67 \text{ mm}}{3.10 \text{ mm}} = 0.8613; \theta_{\text{crit}} = 40.7°$.

Apply Snell's law to the total internal reflection to find the refractive index of the glass:

$n_a \sin\theta_a = n_b \sin\theta_b$ $\quad n_{\text{glass}} \sin\theta_{\text{crit}} = 1.00 \sin 90°$

$$n_{\text{glass}} = \frac{1}{\sin\theta_{\text{crit}}} = \frac{1}{\sin 40.7°} = 1.53$$

EVALUATE: Light incident on the back surface is also totally reflected if it is incident at angles greater than θ_{crit}. If it is incident at less than θ_{crit} it refracts into the air and does not reflect back to the emulsion.

33.48. **IDENTIFY:** Apply Snell's law to the refraction of the light as it passes from water into air.

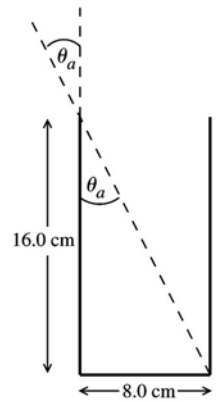
SET UP: $\theta_a = \arctan\left(\dfrac{1.5 \text{ m}}{1.2 \text{ m}}\right) = 51°$. $n_a = 1.00$. $n_b = 1.333$.

EXECUTE: $\theta_b = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.00}{1.333}\sin 51°\right) = 36°.$ Therefore, the distance along the bottom of the pool from directly below where the light enters to where it hits the bottom is $x = (4.0\text{ m})\tan\theta_b = (4.0\text{ m})\tan 36° = 2.9\text{ m}.$ $x_{\text{total}} = 1.5\text{ m} + x = 1.5\text{ m} + 2.9\text{ m} = 4.4\text{ m}.$

EVALUATE: The light ray from the flashlight is bent toward the normal when it refracts into the water.

33.49. **IDENTIFY:** Use Snell's law to determine the effect of the liquid on the direction of travel of the light as it enters the liquid.

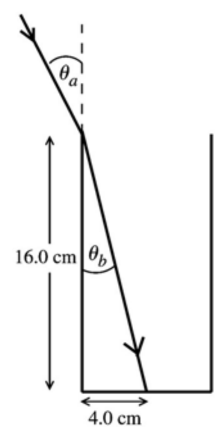
SET UP: Use geometry to find the angles of incidence and refraction. Before the liquid is poured in, the ray along your line of sight has the path shown in Figure 33.49a.



$$\tan\theta_a = \frac{8.0\text{ cm}}{16.0\text{ cm}} = 0.500$$
$$\theta_a = 26.57°$$

Figure 33.49a

After the liquid is poured in, θ_a is the same and the refracted ray passes through the center of the bottom of the glass, as shown in Figure 33.49b.



$$\tan\theta_b = \frac{4.0\text{ cm}}{16.0\text{ cm}} = 0.250$$
$$\theta_b = 14.04°$$

Figure 33.49b

EXECUTE: Use Snell's law to find n_b, the refractive index of the liquid:

$$n_a\sin\theta_a = n_b\sin\theta_b$$
$$n_b = \frac{n_a\sin\theta_a}{\sin\theta_b} = \frac{(1.00)(\sin 26.57°)}{\sin 14.04°} = 1.84$$

EVALUATE: When the light goes from air to liquid (larger refractive index) it is bent toward the normal.

33.50. **IDENTIFY:** The incident angle at the prism $\rightarrow$ water interface is to be the critical angle.

SET UP: The path of the ray is sketched in Figure 33.50. The ray enters the prism at normal incidence so is not bent. For water, $n_{\text{water}} = 1.333$.

EXECUTE: From the figure, $\theta_{\text{crit}} = 45°$. $n_a \sin\theta_a = n_b \sin\theta_b$ gives $n_{\text{glass}} \sin 45° = (1.333)\sin 90°$.

$$n_{\text{glass}} = \frac{1.333}{\sin 45°} = 1.89.$$

EVALUATE: For total internal reflection the ray must be incident in the material of greater refractive index. $n_{\text{glass}} > n_{\text{water}}$, so that is the case here.

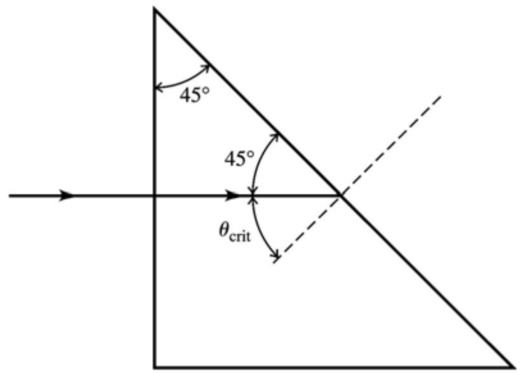


Figure 33.50

33.51. **IDENTIFY:** Apply Snell's law to the water $\rightarrow$ ice and ice $\rightarrow$ air interfaces.

(a) SET UP: Consider the ray shown in Figure 33.51.

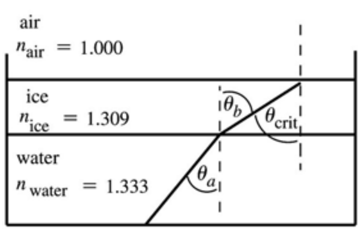


Figure 33.51

We want to find the incident angle θ_a at the water-ice interface that causes the incident angle at the ice-air interface to be the critical angle.

EXECUTE: ice-air interface: $n_{\text{ice}} \sin\theta_{\text{crit}} = 1.0 \sin 90°$

$n_{\text{ice}} \sin\theta_{\text{crit}} = 1.0$ so $\sin\theta_{\text{crit}} = \dfrac{1}{n_{\text{ice}}}$

But from the diagram we see that $\theta_b = \theta_{\text{crit}}$, so $\sin\theta_b = \dfrac{1}{n_{\text{ice}}}$.

water-ice interface: $n_{\text{w}} \sin\theta_a = n_{\text{ice}} \sin\theta_b$

But $\sin\theta_b = \dfrac{1}{n_{\text{ice}}}$ so $n_w \sin\theta_a = 1.0$. $\sin\theta_a = \dfrac{1}{n_{\text{w}}} = \dfrac{1}{1.333} = 0.7502$ and $\theta_a = 48.6°$.

(b) EVALUATE: The angle calculated in part (a) is the critical angle for a water-air interface; the answer would be the same if the ice layer wasn't there!

33.52. **IDENTIFY:** The ray shown in the figure that accompanies the problem is to be incident at the critical angle.

SET UP: $\theta_b = 90°$. The incident angle for the ray in the figure is $60°$.

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$ gives $n_b = \left(\dfrac{n_a \sin\theta_a}{\sin\theta_b}\right) = \left(\dfrac{1.62\sin 60°}{\sin 90°}\right) = 1.40.$

EVALUATE: Total internal reflection occurs only when the light is incident in the material of the greater refractive index.

33.53. **IDENTIFY:** Apply Snell's law to the refraction of each ray as it emerges from the glass. The angle of incidence equals the angle $A = 25.0°$.

SET UP: The paths of the two rays are sketched in Figure 33.53.

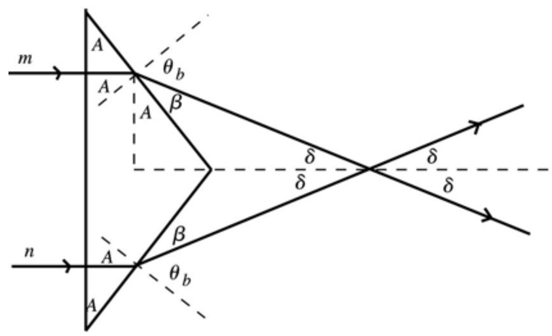


Figure 33.53

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$

$n_{\text{glass}} \sin 25.0° = 1.00 \sin\theta_b$

$\sin\theta_b = n_{\text{glass}} \sin 25.0°$

$\sin\theta_b = 1.66 \sin 25.0° = 0.7015$

$\theta_b = 44.55°$

$\beta = 90.0° - \theta_b = 45.45°$

Then $\delta = 90.0° - A - \beta = 90.0° - 25.0° - 45.45° = 19.55°$. The angle between the two rays is $2\delta = 39.1°$.

EVALUATE: The light is incident normally on the front face of the prism so the light is not bent as it enters the prism.

33.54. **IDENTIFY:** No light enters the gas because total internal reflection must have occurred at the water-gas interface.

SET UP: At the minimum value of S, the light strikes the water-gas interface at the critical angle. We apply Snell's law, $n_a \sin\theta_a = n_{\text{b}} \sin\theta_{\text{b}}$, at that surface.

EXECUTE: **(a)** In the water, $\theta = \dfrac{S}{R} = (1.09\text{ m})/(1.10\text{ m}) = 0.991\text{ rad} = 56.77°$. This is the critical angle.

So, using the refractive index for water from Table 33.1, we get $n = (1.333)\sin 56.77° = 1.12$

(b) (i) The laser beam stays in the water all the time, so

$$t = 2R/v = 2R/\left(\frac{c}{n_{\text{water}}}\right) = \frac{Dn_{\text{water}}}{c} = (2.20\text{ m})(1.333)/(3.00\times 10^8\text{ m/s}) = 9.78\text{ ns}$$

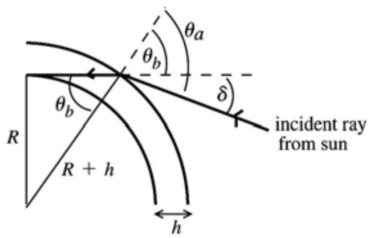
(ii) The beam is in the water half the time and in the gas the other half of the time.

$$t_{\text{gas}} = \frac{Rn_{\text{gas}}}{c} = (1.10\text{ m})(1.12)/(3.00\times 10^8\text{ m/s}) = 4.09\text{ ns}$$

The total time is $4.09\text{ ns} + (9.78\text{ ns})/2 = 8.98\text{ ns}$.

EVALUATE: The gas must be under considerable pressure to have a refractive index as high as 1.12.

33.55. **(a) IDENTIFY:** Apply Snell's law to the refraction of the light as it enters the atmosphere.
SET UP: The path of a ray from the sun is sketched in Figure 33.55.



$\delta = \theta_a - \theta_b$

From the diagram $\sin\theta_b = \dfrac{R}{R+h}$

$\theta_b = \arcsin\left(\dfrac{R}{R+h}\right)$

Figure 33.55

EXECUTE: Apply Snell's law to the refraction that occurs at the top of the atmosphere: $n_a \sin\theta_a = n_b \sin\theta_b$ (a = vacuum of space, refractive, index 1.0; b = atmosphere, refractive index n)

$$\sin\theta_a = n\sin\theta_b = n\left(\frac{R}{R+h}\right) \text{ so } \theta_a = \arcsin\left(\frac{nR}{R+h}\right)$$

$$\delta = \theta_a - \theta_b = \arcsin\left(\frac{nR}{R+h}\right) - \arcsin\left(\frac{R}{R+h}\right)$$

(b) $\dfrac{R}{R+h} = \dfrac{6.38\times10^6\ \text{m}}{6.38\times10^6\ \text{m} + 20\times10^3\ \text{m}} = 0.99688$

$$\frac{nR}{R+h} = 1.0003(0.99688) = 0.99718$$

$$\theta_b = \arcsin\left(\frac{R}{R+h}\right) = 85.47°$$

$$\theta_a = \arcsin\left(\frac{nR}{R+h}\right) = 85.70°$$

$$\delta = \theta_a - \theta_b = 85.70° - 85.47° = 0.23°$$

EVALUATE: The calculated δ is about the same as the angular radius of the sun.

33.56. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** The distance traveled by the light ray is the sum of the two diagonal segments: $d = (x^2 + y_1^2)^{1/2} + ((l-x)^2 + y_2^2)^{1/2}$. Then the time taken to travel that distance is

$$t = \frac{d}{c} = \frac{(x^2 + y_1^2)^{1/2} + ((l-x)^2 + y_2^2)^{1/2}}{c}.$$

(b) Taking the derivative with respect to x of the time and setting it to zero yields

$$\frac{dt}{dx} = \frac{1}{c}\frac{d}{dx}\left[(x^2 + y_1^2)^{1/2} + ((l-x)^2 + y_2^2)^{1/2}\right] = \frac{1}{c}\left[x(x^2 + y_1^2)^{-1/2} - (l-x)((l-x)^2 + y_2^2)^{-1/2}\right] = 0.$$ This gives

$$\frac{x}{\sqrt{x^2 + y_1^2}} = \frac{(l-x)}{\sqrt{(l-x)^2 + y_2^2}}, \quad \sin\theta_1 = \sin\theta_2 \text{ and } \theta_1 = \theta_2.$$

EVALUATE: For any other path between points 1 and 2, that includes a point on the reflective surface, the distance traveled and therefore the travel time is greater than for this path.

33.57. **IDENTIFY** and **SET UP:** Find the distance that the ray travels in each medium. The travel time in each medium is the distance divided by the speed in that medium.

(a) EXECUTE: The light travels a distance $\sqrt{h_1^2 + x^2}$ in traveling from point A to the interface. Along this path the speed of the light is v_1, so the time it takes to travel this distance is $t_1 = \dfrac{\sqrt{h_1^2 + x^2}}{v_1}$. The light travels a distance $\sqrt{h_2^2+(l-x)^2}$ in traveling from the interface to point *B*. Along this path the speed of the light is v_2, so the time it takes to travel this distance is $t_2=\dfrac{\sqrt{h_2^2+(l-x)^2}}{v_2}$. The total time to go from *A* to *B* is $t=t_1+t_2=\dfrac{\sqrt{h_1^2+x^2}}{v_1}+\dfrac{\sqrt{h_2^2+(l+x)^2}}{v_2}$.

(b) $\dfrac{dt}{dx}=\dfrac{1}{v_1}\left(\dfrac{1}{2}\right)(h_1^2+x^2)^{-1/2}(2x)+\dfrac{1}{v_2}\left(\dfrac{1}{2}\right)(h_2^2+(l-x)^2)^{-1/2}2(l-x)(-1)=0$

$$\frac{x}{v_1\sqrt{h_1^2+x^2}}=\frac{l-x}{v_2\sqrt{h_2^2+(l-x)^2}}$$

Multiplying both sides by *c* gives $\dfrac{c}{v_1}\dfrac{x}{\sqrt{h_1^2+x^2}}=\dfrac{c}{v_2}\dfrac{l-x}{\sqrt{h_2^2+(l-x)^2}}$

$\dfrac{c}{v_1}=n_1$ and $\dfrac{c}{v_2}=n_2$ (Eq. 33.1)

From Figure P33.57 in the textbook, $\sin\theta_1=\dfrac{x}{\sqrt{h_1^2+x^2}}$ and $\sin\theta_2=\dfrac{l-x}{\sqrt{h_2^2+(l-x)^2}}$.

So $n_1\sin\theta_1=n_2\sin\theta_2$, which is Snell's law.

EVALUATE: Snell's law is a result of a change in speed when light goes from one material to another.

33.58. **IDENTIFY:** Apply Snell's law to each refraction.

SET UP: Refer to the angles and distances defined in the figure that accompanies the problem.

EXECUTE: **(a)** For light in air incident on a parallel-faced plate, Snell's Law yields:

$n\sin\theta_a=n'\sin\theta_b'=n'\sin\theta_b=n\sin\theta_a'\Rightarrow\sin\theta_a=\sin\theta_a'\Rightarrow\theta_a=\theta_a'$.

(b) Adding more plates just adds extra steps in the middle of the above equation that always cancel out. The requirement of parallel faces ensures that the angle $\theta_n'=\theta_n$ and the chain of equations can continue.

(c) The lateral displacement of the beam can be calculated using geometry:

$d=L\sin(\theta_a-\theta_b')$ and $L=\dfrac{t}{\cos\theta_b'}\Rightarrow d=\dfrac{t\sin(\theta_a-\theta_b')}{\cos\theta_b'}$.

(d) $\theta_b'=\arcsin\left(\dfrac{n\sin\theta_a}{n'}\right)=\arcsin\left(\dfrac{\sin 66.0°}{1.80}\right)=30.5°$ and $d=\dfrac{(2.40\text{ cm})\sin(66.0°-30.5°)}{\cos 30.5°}=1.62\text{ cm}$.

EVALUATE: The lateral displacement in part (d) is large, of the same order as the thickness of the plate.

33.59. **IDENTIFY:** Apply Snell's law to the two refractions of the ray.

SET UP: Refer to the figure that accompanies the problem.

EXECUTE: **(a)** $n_a\sin\theta_a=n_b\sin\theta_b$ gives $\sin\theta_a=n_b\sin\dfrac{A}{2}$. But $\theta_a=\dfrac{A}{2}+\alpha$, so

$\sin\left(\dfrac{A}{2}+\alpha\right)=\sin\dfrac{A+2\alpha}{2}=n\sin\dfrac{A}{2}$. At each face of the prism the deviation is α, so $2\alpha=\delta$ and

$\sin\dfrac{A+\delta}{2}=n\sin\dfrac{A}{2}$.

(b) From part (a), $\delta=2\arcsin\left(n\sin\dfrac{A}{2}\right)-A$. $\delta=2\arcsin\left((1.52)\sin\dfrac{60.0°}{2}\right)-60.0°=38.9°$.

(c) If two colors have different indices of refraction for the glass, then the deflection angles for them will differ:

$$\delta_{\text{red}}=2\arcsin\left((1.61)\sin\frac{60.0°}{2}\right)-60.0°=47.2°$$

$$\delta_{\text{violet}} = 2\arcsin\left((1.66)\sin\frac{60.0°}{2}\right) - 60.0° = 52.2° \Rightarrow \Delta\delta = 52.2° - 47.2° = 5.0°$$

EVALUATE: The violet light has a greater refractive index and therefore the angle of deviation is greater for the violet light.

33.60. **IDENTIFY:** Apply Snell's law and the results of Problem 33.58.

SET UP: From Figure 33.18 in the textbook, $n_r = 1.61$ for red light and $n_v = 1.66$ for violet. In the notation of Problem 33.58, *t* is the thickness of the glass plate and the lateral displacement is *d*. We want the difference in *d* for the two colors of light to be 1.0 mm. $\theta_a = 70.0°$. For red light, $n_a \sin\theta_a = n_b \sin\theta'_b$ gives $\sin\theta'_b = \frac{(1.00)\sin 70.0°}{1.61}$ and $\theta'_b = 35.71°$. For violet light, $\sin\theta'_b = \frac{(1.00)\sin 70.0°}{1.66}$ and $\theta'_b = 34.48°$.

EXECUTE: **(a)** *n* decreases with increasing λ, so *n* is smaller for red than for blue. So beam *a* is the red one.

(b) Problem 33.58 says $d = t\frac{\sin(\theta_a - \theta'_b)}{\cos\theta'_b}$. For red light, $d_r = t\frac{\sin(70° - 35.71°)}{\cos 35.71°} = 0.6938t$ and for violet light, $d_v = t\frac{\sin(70° - 34.48°)}{\cos 34.48°} = 0.7048t$. $d_v - d_r = 1.0$ mm gives $t = \frac{0.10 \text{ cm}}{0.7048 - 0.6938} = 9.1$ cm.

EVALUATE: Our calculation shown that the violet light has greater lateral displacement and this is ray *b*.

33.61. **IDENTIFY** and **SET UP**: The polarizer passes $\frac{1}{2}$ of the intensity of the unpolarized component, independent of ϕ. Out of the intensity I_p of the polarized component the polarizer passes intensity $I_p \cos^2(\phi - \theta)$, where $\phi - \theta$ is the angle between the plane of polarization and the axis of the polarizer.

(a) Use the angle where the transmitted intensity is maximum or minimum to find θ. See Figure 33.61.

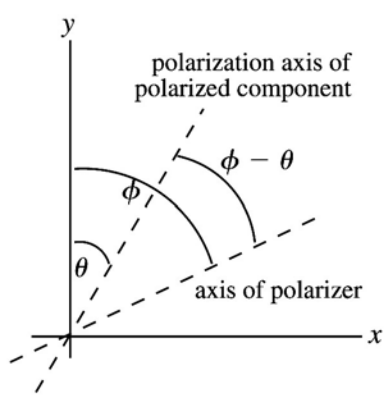


Figure 33.61

EXECUTE: The total transmitted intensity is $I = \frac{1}{2}I_0 + I_p\cos^2(\phi - \theta)$. This is maximum when $\theta = \phi$ and from the table of data this occurs for ϕ between 30° and 40°, say at 35° and $\theta = 35°$. Alternatively, the total transmitted intensity is minimum when $\phi - \theta = 90°$ and from the data this occurs for $\phi = 125°$. Thus, $\theta = \phi - 90° = 125° - 90° = 35°$, in agreement with the above.

(b) IDENTIFY and **SET UP:** $I = \frac{1}{2}I_0 + I_p\cos^2(\phi - \theta)$

Use data at two values of ϕ to determine the two constants I_0 and I_p. Use data where the I_p term is large $(\phi = 30°)$ and where it is small $(\phi = 130°)$ to have the greatest sensitivity to both I_0 and I_p.

EXECUTE: $\phi = 30°$ gives $24.8 \text{ W/m}^2 = \frac{1}{2}I_0 + I_p\cos^2(30° - 35°)$

$24.8\text{ W/m}^2 = 0.500I_0 + 0.9924I_\text{p}$

$\phi = 130°$ gives $5.2\text{ W/m}^2 = \frac{1}{2}I_0 + I_\text{p}\cos^2(130° - 35°)$

$5.2\text{ W/m}^2 = 0.500I_0 + 0.0076I_\text{p}$

Subtracting the second equation from the first gives $19.6\text{ W/m}^2 = 0.9848I_\text{p}$ and $I_\text{p} = 19.9\text{ W/m}^2$. And then $I_0 = 2(5.2\text{ W/m}^2 - 0.0076(19.9\text{ W/m}^2)) = 10.1\text{ W/m}^2$.

EVALUATE: Now that we have I_0, I_p and θ we can verify that $I = \frac{1}{2}I_0 + I_\text{p}\cos^2(\phi - \theta)$ describes that data in the table.

33.62. **IDENTIFY:** The angle by which the plane of polarization of light is rotated depends on the concentration of the compound.

SET UP: If we follow the hint in the problem and graph (not shown) the concentration *C* as a function of the rotation angle ϑ, *l*-leucine and *d*-glutamic acid both exhibit linear relationships between *C* and ϑ, with the *y*-intercept being zero in both cases. Using the slope *y*-intercept form of the equation of a straight line $(y = mx + b)$, we can find the equation for *C* as a function of ϑ.

EXECUTE: For *l*-leucine, the slope of the graph is $m = -9.09\dfrac{\text{g}}{100\text{ mL}}\cdot\text{deg}^{-1}$, so the equation for *C* as a function of ϑ is $C = -\left(9.09\dfrac{\text{g}}{100\text{ mL}}\cdot\text{deg}^{-1}\right)\vartheta$. For *d*-glutamic acid, the slope is $m = 8.06\dfrac{\text{g}}{100\text{ mL}}\cdot\text{deg}^{-1}$, so the desired equation is $C = \left(8.06\dfrac{\text{g}}{100\text{ mL}}\cdot\text{deg}^{-1}\right)\vartheta$. The opposite signs in the equations tell us that the two compounds rotate the plane of polarization in opposite directions.

EVALUATE: Inspection of the data indicates that the slope is constant and that the *y*-intercept is zero (no concentration, no rotation). We can use data points to find the slope. For example, using the second and third data points for *l*-leucine, the slope is

$$m = \frac{\Delta C}{\Delta\vartheta} = \frac{5.0\text{ g/(100 mL)} - 2.0\text{ g/(100 mL)}}{-0.55° - (-0.22°)} = \frac{3.0\text{ g/(100 mL)}}{-0.33°} = -9.09\frac{\text{g}}{100\text{ mL}}\cdot\text{deg}^{-1},$$

which is the same result we got from the graph, leading to the same equation.

33.63. **IDENTIFY:** The reflected light is totally polarized when light strikes a surface at Brewster's angle.

SET UP: At the plastic wall, Brewster's angle obeys the equation $\tan\theta_\text{p} = n_b/n_a$, and Snell's law, $n_a\sin\theta_a = n_b\sin\theta_b$, applies at the air-water surface.

EXECUTE: To be totally polarized, the reflected sunlight must have struck the wall at Brewster's angle. $\tan\theta_\text{p} = n_b/n_a = (1.61)/(1.00)$ and $\theta_\text{p} = 58.15°$.

This is the angle of incidence at the wall. A little geometry tells us that the angle of incidence at the water surface is $90.00° - 58.15° = 31.85°$. Applying Snell's law at the water surface gives

$$(1.00)\sin 31.85° = 1.333\sin\theta \text{ and } \theta = 23.3°$$

EVALUATE: We have two different principles involved here: Reflection at Brewster's angle at the wall and Snell's law at the water surface.

33.64. **IDENTIFY:** The number of wavelengths in a distance *D* of material is D/λ, where λ is the wavelength of the light in the material.

SET UP: The condition for a quarter-wave plate is $\dfrac{D}{\lambda_1} = \dfrac{D}{\lambda_2} + \dfrac{1}{4}$, where we have assumed $n_1 > n_2$ so $\lambda_2 > \lambda_1$.

EXECUTE: **(a)** $\dfrac{n_1 D}{\lambda_0} = \dfrac{n_2 D}{\lambda_0} + \dfrac{1}{4}$ and $D = \dfrac{\lambda_0}{4(n_1 - n_2)}$.

(b) $D = \dfrac{\lambda_0}{4(n_1 - n_2)} = \dfrac{5.89\times10^{-7}\text{ m}}{4(1.875 - 1.635)} = 6.14\times10^{-7}\text{ m}.$

EVALUATE: The thickness of the quarter-wave plate in part (b) is 614 nm, which is of the same order as the wavelength in vacuum of the light.

33.65. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: $\cos(\alpha-\beta)=\sin\alpha\sin\beta+\cos\alpha\cos\beta$. $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.

EXECUTE: **(a)** Multiplying Eq. (1) by $\sin\beta$ and Eq. (2) by $\sin\alpha$ yields:

(1): $\dfrac{x}{a}\sin\beta=\sin\omega t\cos\alpha\sin\beta-\cos\omega t\sin\alpha\sin\beta$ and (2): $\dfrac{y}{a}\sin\alpha=\sin\omega t\cos\beta\sin\alpha-\cos\omega t\sin\beta\sin\alpha$.

Subtracting yields: $\dfrac{x\sin\beta-y\sin\alpha}{a}=\sin\omega t(\cos\alpha\sin\beta-\cos\beta\sin\alpha)$.

(b) Multiplying Eq. (1) by $\cos\beta$ and Eq. (2) by $\cos\alpha$ yields:

(1): $\dfrac{x}{a}\cos\beta=\sin\omega t\cos\alpha\cos\beta-\cos\omega t\sin\alpha\cos\beta$ and (2): $\dfrac{y}{a}\cos\alpha=\sin\omega t\cos\beta\cos\alpha-\cos\omega t\sin\beta\cos\alpha$.

Subtracting yields: $\dfrac{x\cos\beta-y\cos\alpha}{a}=-\cos\omega t(\sin\alpha\cos\beta-\sin\beta\cos\alpha)$.

(c) Squaring and adding the results of parts (a) and (b) yields:

$$(x\sin\beta-y\sin\alpha)^2+(x\cos\beta-y\cos\alpha)^2=a^2(\sin\alpha\cos\beta-\sin\beta\cos\alpha)^2$$

(d) Expanding the left-hand side, we have:

$$x^2(\sin^2\beta+\cos^2\beta)+y^2(\sin^2\alpha+\cos^2\alpha)-2xy(\sin\alpha\sin\beta+\cos\alpha\cos\beta)$$
$$=x^2+y^2-2xy(\sin\alpha\sin\beta+\cos\alpha\cos\beta)=x^2+y^2-2xy\cos(\alpha-\beta).$$

The right-hand side can be rewritten: $a^2(\sin\alpha\cos\beta-\sin\beta\cos\alpha)^2=a^2\sin^2(\alpha-\beta)$. Therefore,

$x^2+y^2-2xy\cos(\alpha-\beta)=a^2\sin^2(\alpha-\beta)$. Or, $x^2+y^2-2xy\cos\delta=a^2\sin^2\delta$, where $\delta=\alpha-\beta$.

EVALUATE: **(e)** $\delta=0: x^2+y^2-2xy=(x-y)^2=0\Rightarrow x=y$, which is a straight diagonal line

$\delta=\dfrac{\pi}{4}: x^2+y^2-\sqrt{2}xy=\dfrac{a^2}{2}$, which is an ellipse

$\delta=\dfrac{\pi}{2}: x^2+y^2=a^2$, which is a circle. This pattern repeats for the remaining phase differences.

33.66. **IDENTIFY:** Apply Snell's law to each refraction.

SET UP: Refer to the figure that accompanies the problem.

EXECUTE: **(a)** By the symmetry of the triangles, $\theta_b^A=\theta_a^B$, and $\theta_a^C=\theta_r^B=\theta_a^B=\theta_b^A$. Therefore,

$\sin\theta_b^C=n\sin\theta_a^C=n\sin\theta_b^A=\sin\theta_a^A=\theta_b^C=\theta_a^A$.

(b) The total angular deflection of the ray is $\Delta=\theta_a^A-\theta_b^A+\pi-2\theta_a^B+\theta_b^C-\theta_a^C=2\theta_a^A-4\theta_b^A+\pi$.

(c) From Snell's Law, $\sin\theta_a^A=n\sin\theta_b^A\Rightarrow\theta_b^A=\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)$.

$\Delta=2\theta_a^A-4\theta_b^A+\pi=2\theta_a^A-4\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)+\pi$.

(d) $\dfrac{d\Delta}{d\theta_a^A}=0=2-4\dfrac{d}{d\theta_a^A}\left(\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)\right)\Rightarrow 0=2-\dfrac{4}{\sqrt{1-\dfrac{\sin^2\theta_1}{n^2}}}\cdot\left(\dfrac{\cos\theta_1}{n}\right)\cdot 4\left(1-\dfrac{\sin^2\theta_1}{n^2}\right)=\left(\dfrac{16\cos^2\theta_1}{n^2}\right)$.

$4\cos^2\theta_1=n^2-1+\cos^2\theta_1$. $3\cos^2\theta_1=n^2-1$. $\cos^2\theta_1=\dfrac{1}{3}(n^2-1)$.

(e) For violet: $\theta_1=\arccos\left(\sqrt{\dfrac{1}{3}(n^2-1)}\right)=\arccos\left(\sqrt{\dfrac{1}{3}(1.342^2-1)}\right)=58.89°$.

$\Delta_{\text{violet}}=139.2°\Rightarrow\theta_{\text{violet}}=40.8°$.

For red: $\theta_1 = \arccos\left(\sqrt{\frac{1}{3}(n^2 - 1)}\right) = \arccos\left(\sqrt{\frac{1}{3}(1.330^2 - 1)}\right) = 59.58°.$

$\Delta_{\text{red}} = 137.5° \Rightarrow \theta_{\text{red}} = 42.5°.$

EVALUATE: The angles we have calculated agree with the values given in Figure 33.20d in the textbook. θ_1 is larger for red than for violet, so red in the rainbow is higher above the horizon.

33.67. **IDENTIFY:** Follow similar steps to Challenge Problem 33.66.

SET UP: Refer to Figure 33.20e in the textbook.

EXECUTE: **(a)** The total angular deflection of the ray is

$\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_b^A + \pi - 2\theta_b^A + \theta_a^A - \theta_b^A = 2\theta_a^A - 6\theta_b^A + 2\pi,$ where we have used the fact from the previous problem that all the internal angles are equal and the two external equals are equal. Also using the Snell's law relationship,

we have: $\theta_b^A = \arcsin\left(\frac{1}{n}\sin\theta_a^A\right)$. $\Delta = 2\theta_a^A - 6\theta_b^A + 2\pi = 2\theta_a^A - 6\arcsin\left(\frac{1}{n}\sin\theta_a^A\right) + 2\pi.$

(b) $\frac{d\Delta}{d\theta_a^A} = 0 = 2 - 6\frac{d}{d\theta_a^A}\left(\arcsin\left(\frac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \frac{6}{\sqrt{1 - \frac{\sin\theta_2}{n^2}}} \cdot \left(\frac{\cos\theta_2}{n}\right).$

$n^2\left(1 - \frac{\sin^2\theta_2}{n^2}\right) = (n^2 - 1 + \cos^2\theta_2) = 9\cos^2\theta_2.\ \cos^2\theta_2 = \frac{1}{8}(n^2 - 1).$

(c) For violet, $\theta_2 = \arccos\left(\sqrt{\frac{1}{8}(n^2 - 1)}\right) = \arccos\left(\sqrt{\frac{1}{8}(1.342^2 - 1)}\right) = 71.55°.$ $\Delta_{\text{violet}} = 233.2°$ and

$\theta_{\text{violet}} = 53.2°.$

For red, $\theta_2 = \arccos\left(\sqrt{\frac{1}{8}(n^2 - 1)}\right) = \arccos\left(\sqrt{\frac{1}{8}(1.330^2 - 1)}\right) = 71.94°.$ $\Delta_{\text{red}} = 230.1°$ and $\theta_{\text{red}} = 50.1°.$

EVALUATE: The angles we calculated agree with those given in Figure 33.20e in the textbook. The color that appears higher above the horizon is violet. The colors appear in reverse order in a secondary rainbow compared to a primary rainbow.